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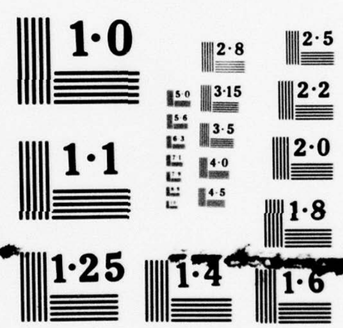
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SECONDARY FLOW VORTICITY IN THE PASSAGE OF A ROTOR

M. L. Billet

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Subject: Secondary Flow Vorticity in the Passage of a Rotor

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List of Symbols

$a_{b'}$	distance between neighboring streamlines in b' direction
$a_{n'}$	distance between neighboring streamlines in n' direction
W	relative velocity
$\vec{\Omega}$	rotation vector
$\vec{\xi}$	relative vorticity vector ($\vec{\xi}=\nabla' x\vec{W}=\vec{\omega}-2\vec{\Omega}$)
R'	radius of curvature of relative streamline
τ	radius of torsion of relative streamline
s'	relative streamwise direction (flow direction)
n'	principle normal direction of relative streamline defined as positive toward center of curvature of the streamlines
b'	bi-normal direction of relative streamline ($\vec{s}' x \vec{n}'=\vec{b}'$)
$\Omega_{b'}$	component of rotation vector in bi-normal direction
$\Omega_{n'}$	component of rotation vector in normal direction
$\omega_{b'}$	component of absolute vorticity vector in bi-normal direction ($\omega_{b'}=W/R'$)
$\omega_{n'}$	component of absolute vorticity vector in normal direction ($\omega_{n'}=\partial W/\partial b'$)
$\omega_{s'}$	component of absolute vorticity vector in streamwise direction
δ	boundary layer thickness
ℓ	blade span
c	blade chord
Ω	magnitude of rotation vector
r_o	rotor inner radius
R_R	rotor tip radius
ϕ	flow coefficient ($\phi=V_\infty/U_{TIP}$)
Φ	blade stagger angle
β	relative rotor flow angle

List of Symbols (Cont.)

Ψ	rake angle of the rotor
ε'	turning angle of the relative streamline
γ	angle between the bi-normal direction and the normal to the stream surfaces
V_x	axial velocity
V_∞	free stream velocity
Γ	blade circulation

Subscripts

1	upstream of rotor
2	downstream of rotor

Primed Symbols signify that terms are expressed in relative coordinate system.

List of Figures

[1] INTRODUCTION

In order to accomplish work in any rotor, angular momentum must be imparted to the fluid. The lifting action of the rotor blades produces cross channel pressure gradients. At a sufficient distance from the inner wall, viscous effects are negligible and the pressure gradients are balanced by streamline curvature. Close to the inner wall, a boundary layer exists. The fluid within this boundary layer does not have sufficient momentum to balance the pressure gradients imposed by the inviscid outer flow. The result is a cross-flow component containing vorticity aligned in the streamwise direction. This additional streamwise vorticity causes a deviation in the rotor outlet angles. In some cases, this streamwise vorticity causes a roll-up of the low momentum fluid near the wall into a vortex-type flow [1]*.

There are no simple techniques that accurately predict these inner wall velocity gradient effects for rotational flows. It is important to recognize that a satisfactory description of this boundary layer flow cannot come from refinements of the two-dimensional boundary layer theory, but rather from three-dimensional analysis. The reason is that boundary layer behavior in rotors exhibit variations in its lateral direction. The occurrence of these secondary flows which are boundary layer flows having a component normal to the mainstream direction arise principally from blade to blade and radial pressure gradients. Similarity between the results of secondary flow analysis and the cross flow in the outer part of a three-dimensional boundary layer has been established by Horlock [2].

The distributed passage secondary vorticity generated by turning a boundary layer flow is one of three possible types of secondary streamwise vorticity,

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*Numbers in brackets refer to documents in the list of references.

which can occur near the inner of a rotor. As discussed by Hawthorne [3], three components of secondary vorticity are identified in the direction of flow at the exit of a blade row. These secondary sources of vorticity are usually regarded as a perturbation on the primary flow. They are (1) a distributed passage vorticity in the blade passage which may result in the formation of the so-called passage vortex, (2) the trailing shed vorticity, and (3) the trailing filament vorticity. The latter two types of vorticity are due to the vortex sheet leaving the blade trailing edge and lead to the formation of another vortex which is opposite in rotation to that of the passage vortex as shown in Figure 1. The trailing filament vorticity is caused by the stretching of the vortex filaments as they move over the surfaces of the blades. The last component of vorticity is the trailing shed vorticity which is caused by the variation of circulation along the span of the blades.

With the exception of trailing shed circulation which exists along the blade even when the incoming flow is uniform, each of these vorticity components could be attributed to the existence of the wall boundary layer and not the change of the boundary layer due to viscous effects as flow passes through the rotor. Therefore, the primary assumption leading to the existing theoretical descriptions of secondary flows is that viscous effects produce a boundary layer on the wall upstream of the rotor. Whereas within the rotor, the imposed pressure gradients play the major role and viscosity has little effect on the resulting secondary flows. This assumption is characteristic of what is generally termed inviscid secondary flow analysis.

As shown in an analysis of secondary flows by Came and Marsh [4], the total strength of these three secondary components of streamwise

vorticity is zero in the flow downstream of a many bladed cascade. However, this vorticity in the flow does have an effect on the flow field. The primary effect of this secondary vorticity is a deviation in the blade outlet angle due to the passage secondary vorticity. This deviation can be quite large when the incoming velocity gradient to the rotor is large or when the flow is turned through a large angle. The solution for this deviation must be consistent with the trailing vortex sheet, but the strength of this sheet does not need to be known.

Another effect of secondary distributed vorticity generated near the wall of a rotor is evidenced by the existence of a cavitating vortex. The appearance of the cavitating vortex is similar to that shown in Figure 2. The structure of the vortex varies with velocity gradient-rotor configuration. The magnitude of the minimum pressure coefficient ($C_{P_{min}}$) associated with this vortex depends on both the primary and secondary vorticity. However, the resultant strongly swirling flow appears to be organized by the secondary vorticity.

Theoretical understanding of secondary flows through a rotor is aided by approximate solutions of the fluid flow equations that govern the flow process. The equations themselves are approximate because certain simplifying assumptions must be made before the solution can be obtained. However, any secondary flow theory which is applicable to the vortex problem must be least include the effects of blade twist, variable relative velocity through the rotor, large velocity gradients and rotation.

The objective of this report is to review and discuss secondary flow theories which can be employed to estimate the vorticity created near the

inner wall of a rotor. There are many different approaches to the derivation of secondary flow equations; however, Horlock [5] in a discussion of a paper on secondary vorticity in axial compressor blade rows shows that the different approaches all lead to essentially the same result. A helpful review of secondary flow theories can be found in the papers of Lakshminarayana and Horlock [6], the staff of NASA Lewis Research Center [7], Hawthorne and Novak [8], Lakshminarayana and Horlock [9] and Salvage [10].

[2] PASSAGE SECONDARY VORTICITY IN A ROTOR

The equation of motion for incompressible flow with reference to axis rotating at constant angular velocity $(\vec{\Omega})$ is given in Greenspan [11] as

$$\vec{\xi} x \vec{W} + 2\vec{\Omega} x \vec{W} = - \nabla'\left(\frac{P}{\rho}\right) - \nabla'[W^2/2 - 1/2(\vec{\Omega} x \vec{r}) \cdot (\vec{\Omega} x \vec{r})] - (\mu/\rho)\nabla' x \nabla' x \vec{W} \quad (1)$$

where the prime denotes differentiation with respect to the rotating frame $(\vec{\Omega})$, $\vec{W}$ is the relative velocity and $\vec{\xi}$ is the relative vorticity given by, $\vec{\xi}=\nabla' x \vec{W}=\vec{\omega}-2\vec{\Omega}$. Taking the curl of Equation (1) gives the vorticity equation

$$(\vec{W}\cdot\nabla')\vec{\xi} = (\vec{\xi}\cdot\nabla')\vec{W} - 2\nabla' x(\vec{\Omega} x \vec{W}) + (\mu/\rho)\nabla'^2\vec{\xi} \quad . \quad (2)$$

Expessions for the absolute secondary vorticity (ω) defined along a relative streamline valid for a rotor are obtained from Equation (2) by Lakshminarayana and Horlock [9]. The resultant equations are

$$\frac{\partial}{\partial s'}\left[\frac{\omega_{s'}}{W}\right] = \frac{2\omega_{n'}}{WR'} + \frac{2\Omega_{b'}\omega_{n'}}{W^2} - \frac{2\Omega_{n'}\omega_{b'}}{W^2} + (\mu/\rho)\vec{s}' \cdot \frac{\nabla'^2\xi}{W^2} \quad . \quad (3)$$

$$\frac{\partial}{\partial n'}[\omega_{n'}W] = \frac{W\omega_{b'}}{\tau} - \frac{W\omega_{n'}}{a_{b'}}\frac{\partial a_{b'}}{\partial s'} + (\mu/\rho)\vec{n}\cdot\nabla'^{2}\xi \tag{4}$$

where as shown in Figure 3, s', n', b' represent the natural coordinates for the relative flow, W is the relative velocity, $\omega_{s'}$ and $\omega_{n'}$ are absolute vorticity resolved along the relative streamline (s') and the principal normal direction (n'), Ω is the rotation vector and R' is the radius of curvature of the relative streamline. In this relative coordinate system indicated by the primes, the streamwise direction ($\vec{s}'$) is defined along the flow direction, the principal normal direction ($\vec{n}'$) is defined toward the center of curvature of the steamlines, and the bi-normal direction ($\vec{b}'$) is defined as $\vec{s}' \times \vec{n}' = \vec{b}'$. Further definitions and descriptions of the flow in this relative coordinate system are given by Lakshminarayana and Horlock [9].

The means by which the streamwise component of vorticity is produced in this relative flow are similar to those in a stationary system. However, it is important to note that additional secondary vorticity is generated when $\vec{\Omega} \times \vec{W}$ has a component in the relative streamwise direction. Rotation has no effect when the absolute vorticity vector lies in the s'-n' plane and the rotation ($\vec{\Omega}$) has no component in the bi-normal direction ($\vec{b}'$).

The equations for secondary vorticity created in the passage namely Equations (3) and (4) are extremely difficult to apply because they are partial differential equations. Therefore, some simplifying assumptions and specification of the flow field must be made for each application. The basic flow parameters for this case discussed in this paper are a thick boundary layer entering the rotor and the rotor operating at a flow coefficient near 1.0. It is also important to note that the primary flow through the rotor is rotational.

In order to evaluate the relative importance of the terms that describe the passage secondary vorticity, the vorticity equations were first nondimensionalized. Then an order of magnitude evaluation can be used in the selection of the terms.

The normal vorticity equation can now be evaluated in the following manner:

$$\frac{W}{W_o} = \overline{W} \qquad \frac{a_{b'}}{n\ell} = \overline{a}_{b'}$$

$$\frac{\omega_{n'}}{W_o/\delta} = \overline{\omega}_{n'} \qquad \frac{s'}{c} = \overline{s}'$$

$$\frac{\omega_{b'}}{W_o/R'_m} = \overline{\omega}_{b'} \qquad \frac{\tau}{R_R} = \overline{\tau}$$

where

W_o ∿ initial relative velocity

δ ∿ boundary layer thickness

R'_m ∿ camber radius at mid-span

$a_{b'}$ ∿ distance between neighboring streamlines in b' direction

ℓ ∿ blade span

R_R ∿ rotor tip radius

r_o ∿ rotor inner radius.

Applying these nondimensional variables to Equation (4) and ignoring viscous terms in the normal direction, we find

$$\frac{W_o^{\,2}}{\delta \cdot c} \cdot \frac{\partial}{\partial \bar{s}'} (\bar{\omega}_{n'}, \bar{W}) = \frac{W_o^{\,2}}{R_m' \cdot r} \left\{ \frac{\bar{W}\,\bar{\omega}_{b'}}{\bar{\tau}} \right\} - \frac{W_o^{\,2}}{\delta \cdot c} \left\{ \frac{\bar{W}\,\bar{\omega}_{n'}}{\bar{a}_{b'}} \frac{\partial \bar{a}_{b'}}{\partial \bar{s}'} \right\} \tag{5-a}$$

or

$$\frac{\partial}{\partial \bar{s}'} \{\omega_{n'}, \bar{W}\} = \frac{\delta \cdot c}{R_m' \cdot r} \left\{ \frac{\bar{W}\,\bar{\omega}_{b'}}{\bar{\tau}} \right\} - \frac{\bar{W}\,\bar{\omega}_{n'}}{\bar{a}_{b'}} \cdot \frac{\partial \bar{a}_{b'}}{\partial \bar{s}'} \quad . \tag{5-b}$$

The ratio of boundary layer thickness to rotor tip radius is of order one for the applications considered in this paper. Also, the blade chord (c) is an order to magnitude less than the camber radius. Therefore, the normal relative vorticity equation becomes

$$\frac{\partial}{\partial s'} (\omega_{n'}, W) = - \frac{W\,\omega_{n'}}{a_{b'}} \frac{\partial a_{b'}}{\partial s'} \quad . \tag{6}$$

Equation (6) can be integrated by combining the two terms into one differential. The result is

$$\omega_{n'} = \omega_{n_1'} \frac{W_1\, a_{b_1}'}{W\, a_{b'}} \tag{7}$$

where the subscript 1 refers to the rotor inlet and unnumbered subscripts refer to any position along a streamline in the rotor. This simplified equation shows a dependence of the relative component of the normal vorticity on the variation of relative velocity (W) along a streamline, on the relative component of the absolute inlet normal vorticity ($\omega_{n_1'}$) and on the flow convergence-divergence in the bi-normal direction ($a_{b'}$).

Equation (7) can be evaluated along a relative streamline if the streamline spacing ($a_{b'}$) in the bi-normal direction is known. This can be easily estimated by applying a numerical technique, such as the

streamline curvature method, to the primary flow field. For some rotors having a high hub to tip ratio, the streamline spacing in the radial direction ($a_{b'}$) remains constant and Equation (7) then becomes

$$\omega_{n'} = \omega_{n'_1} \cdot \frac{W_1}{W} \quad . \tag{8}$$

This result relates the change in the relative component of the absolute normal vorticity ($\omega_{n'}$) to changes in the relative velocity (W). Also, assuming a weak shear flow so that the axial velocity remains constant gives

$$\omega_{n'} = \omega_{n'_1} \cdot \frac{\cos \beta_1}{\cos \beta} \quad . \tag{9}$$

The last two relationships are commonly used for secondary flow cascade theory.

In a similar manner, the streamwise secondary vorticity equation can be nondimensionalized and evaluated as follows:

$$\frac{\omega_{s'}}{\frac{c}{R'_m} \cdot \frac{W_o}{\delta}} = \bar{\omega}_{s'} \qquad \frac{b'}{\delta} = \bar{b}'$$

$$\frac{\Omega_b}{\omega \sin \Psi} = \bar{\Omega}_{b'} \qquad \frac{\Omega_{n'}}{\bar{\Omega} \sin \phi} = \bar{\Omega}_{n'}$$

$$\frac{R'}{R'_m} = \bar{R}'$$

where

Ψ $\sim$ rake angle of the rotor

Ω $\sim$ magnitude of rotational vector of rotor.

Applying these additional nondimensional variables to Equation (3) for the streamwise vorticity we get

$$\frac{\delta}{\partial \overline{s}'}\left\{\frac{\overline{\omega}_{s'}}{\overline{W}}\right\} = \frac{2\overline{\omega}_{n'}}{\overline{W}\,\overline{R}'} + \frac{R'_m\,\Omega\,\sin\Psi}{W_o}\left\{\frac{2\,\overline{\Omega}_{b'}\,\overline{\omega}_{n'}}{\overline{W}^2}\right\} - \left\{\frac{2\,\overline{\Omega}_{n'}\,\omega_{b'}}{\overline{W}^2}\right\}$$

$$+\frac{c}{\delta}\cdot\frac{1}{Re}\left\{\frac{1}{\overline{W}^2}\,\frac{\partial^2\,\overline{\omega}_{s'}}{\partial\,\overline{b}'^2}\right\} \quad . \tag{10}$$

The second term on the right-hand side of Equation (10) is of order one because the flow coefficient is of order one. This term is due to curvature induced secondary vorticity. The third term is also of order one and is due to rotation-induced secondary vorticity. However, both of these terms are as important to secondary flow development as the first term.

The fourth term on the right-hand-side of Equation (10) is a viscous dissipation term which can be neglected in most cases. Even if the Reynolds number is based on local parameters in the boundary layer, this term is still small compared to the other terms. This result is due to the boundary layer thickness being the same order as the rotor tip radius and much larger than the blade chord. These gradients are not as large as would be the case if the boundary layer thickness would be much less than the rotor tip radius. However, this viscous term does approach one near the viscous sublayer.

The rate of change of the streamwise vorticity from Equation (10) simplifies to

$$\frac{\partial}{\partial s'}\left(\frac{\omega_{s'}}{W}\right) = \frac{2\omega_{n'}}{WR'} + \frac{2\Omega_{b'}\omega_{n'}}{W^2} - \frac{2\Omega_{n'}\omega_{b'}}{W^2} \quad . \tag{11}$$

Integrating Equation (11) in the streamwise direction yields

$$\frac{\omega_{s_2'}}{W_2} - \frac{\omega_{s_1'}}{W_1} = \int_1^2 \frac{2\omega_{n'}ds'}{WR'} + \int_1^2 \frac{2\Omega_{b'}\omega_{n'}ds}{W^2} - \int_1^2 \frac{2\Omega_{n'}\omega_{b'}ds}{W^2} \tag{12-a}$$

where $\omega_{b'} \sim W/R'$.

Solving Equation (12-a) for $\omega_{s_2'}$ one finds that

$$\omega_{s_2'} = W_2 \int_1^2 \frac{2\omega_{n'}}{WR'}\, ds' + W_2 \int_1^2 \frac{2\Omega_{b'}\omega_{n'}}{W^2}\, ds' - W_2 \int_1^2 \frac{2\Omega_{n'}\omega_{b'}}{W^2}\, ds' + \omega_{s_1'}\left\{\frac{W_2}{W_1}\right\} \quad . \tag{12-b}$$

If the nonuniform flow through the rotor was such that the relative streamlines and the absolute vortex lines lie on cylindrical surfaces ($\omega_{b'}=0$) and the rotation vector ($\vec{\Omega}$) is parallel to the axis of the cylindrical surfaces, then Equation (12-a) would become

$$\frac{\omega_{s_2'}}{W_2} - \frac{\omega_{s_1'}}{W_1} = \int_1^2 \frac{2\omega_{n'}}{WR'}\, ds' \quad . \tag{13}$$

This relationship is commonly used in most cascade flow calculations.

In summary for a rotor having the following characteristics:

1. low hub-to-tip ratio,
2. rotor operating flow coefficient of approximately one
3. nonuniform flow over the span of the blade,

the following two equations are needed to estimate the passage secondary flow:

$$\omega_{n'} = \omega_{n_1'} \cdot \frac{W_1 \, a_{b_1'}}{W \, a_{b'}} \tag{7}$$

$$\omega_{s_2'} = W_2 \int_1^2 \frac{2\omega_{n'}}{WR'} ds' + W_2 \int_1^2 \frac{2\Omega_{b'}\omega_{n'}}{W^2} ds' - W_2 \int_1^2 \frac{2\Omega_{n'}\omega_{b'}}{W^2} ds' + \omega_{s_1'} \left\{\frac{W_2}{W_1}\right\} . \tag{12-b}$$

[3] OTHER APPROXIMATE FORMS FOR THE STREAMWISE VORTICITY

It is important in secondary flow analysis to understand the flow field in order to determine which vorticity terms are important. Numerous analytical and semi-analytical relationships have been used to calculate the streamwise vorticity.

Many investigators [12-16] have derived expressions for cascade secondary vorticity. The basic analysis is due to Squire and Winter [12]. Their expression assumes that the flow is incompressible and inviscid. Also, they assume a constant velocity along the streamlines which implies that the normal component of vorticity remains constant. Their result for a rotating frame of reference is

$$\omega_{s_2'} - \omega_{s_1'} = 2\omega_{n'} \, \varepsilon' \tag{14}$$

where ε' is the turning angle of the relative streamline.

Hawthorne [13] developed a more general theory for the secondary flow. For an inviscid and incompressible fluid the equation written for a rotating frame of reference would be

$$\left(\frac{\omega_{s'}}{W}\right)_2 - \left(\frac{\omega_{s'}}{W_1}\right) = 2\int_1^2 \frac{\left|\frac{\nabla' I}{\rho}\right| \cos\gamma}{W^2} d\varepsilon' \qquad (15)$$

where $\nabla' I/\rho = \vec{W}x\vec{\omega}$ and γ is the angle between the bi-normal direction and the normal to the Bernoulli planes or stream surfaces. The validity of such an analysis has been confirmed by several experimental investigators for a stationary cascade.

Loos and Zwaanweld [14] have tried to simplify Equation (15) by assuming $\omega_{s'_1}=0$ and $\cos\gamma=1$. This is a key assumption which implies that the stream surface are surfaces of revolution. However, it is very unlikely that stream surfaces in any real cascade are surfaces of revolution. Their equivalent expression in a rotating frame of reference is

$$\omega_{s'_2} = -\frac{\omega_{n'_1}}{\cos\beta_1 \cos\beta_2}\left[(\beta_2-\beta_1) + \frac{\sin 2\beta_2 - \sin 2\beta_1}{2}\right] . \qquad (16)$$

This equation allows only for a constant axial velocity through the rotor and a small angle of turning.

Marsh [15] has extended this result to include the effect of a change in axial velocity on the secondary flow at exit from a cascade. It is shown that for a row of inlet guide vanes the change of axial velocity across the blade row has a significant effect on the secondary vorticity. His expression in an equivalent rotating frame would be

$$\omega_{s'_2} = \frac{V_{x_2}}{V_{x_1}} \frac{\cos\beta_1}{\cos\beta_2} \omega_{s'_1} + \frac{\omega_{n'_1}}{\cos\beta_1 \cos\beta_2}\left[(1/2 \sin 2\beta_2\right.$$

$$-\frac{V_{x_2}}{2V_{x_1}}\sin 2\beta_1) - (\beta_2-\beta_1)\left[\frac{\frac{V_{x_2}}{V_{x_1}}\tan\beta_2-\tan\beta_1}{\tan\beta_2-\tan\beta_1}\right] \quad . \tag{17}$$

Lakshminarayana and Horlock [16] have extended Equation (15) to include the effect of stream surface (Bernoulli surface) rotation on the strength of secondary vorticity. They used an expression by Dean [17] in which the direction and magnitude of the warping of real stream surfaces is related to the blade geometry. This expression in terms of a rotating coordinate system is

$$\gamma = \frac{\pi}{2} - \frac{c}{2W\varepsilon'_{12}}\frac{dW}{db'}\varepsilon'^2 \tag{18}$$

where the streamlines are assumed to follow the curvature of the blade camber line. As can be seen from Equation (18), stream surface rotation becomes important when the incoming velocity gradients $(\frac{dW}{db'})$ are large and when the flow is turned through a large angle (ε').

Using Equation (18) in Equation (15) gives their equation for the streamwise vorticity assuming a constant axial velocity and the stagnation pressure varies only in the spanwise direction. Again, the equivalent expression for a rotating reference frame is

$$\left(\frac{\omega_{s'}}{W}\right)_2 - \left(\frac{\omega_{s'}}{W}\right)_1 = \frac{2\omega_{n'_1}}{\cos\beta_1\cos\beta_2}\left[\frac{\sin 2\beta_2 - \sin 2\beta_1}{2} + \frac{\beta_2-\beta_1}{2}\right.$$

$$-\frac{1}{8}\left(\frac{c}{W\varepsilon'}\omega_{n'_1}\right)^2 \left\{\frac{\beta_2^5-\beta_1^5}{10} + \frac{\beta_2^4\sin 2\beta_2-\beta_1^4\sin 2\beta_1}{4}\right.$$

$$- \beta_2^3\cos^2\beta_2-\beta_1^3\cos^2\beta_1 - \frac{\beta_2^3-\beta_1^3}{2} - \frac{3}{4}(\beta_2\cos2\beta_2-\beta_1\cos2\beta_1)$$

$$\left. \left. - \frac{3}{4}(\beta_2^2\sin2\beta_2-\beta_1^2\sin2\beta_1) + \frac{3}{8}(\sin2\beta_2-\sin2\beta_1)\right\}\right] \quad . \tag{19}$$

Dixon [18] developed an equation which can be applied to calculate the secondary flow in an annular cascade having a low hub/tip radius ratio. In his analysis, vortex filaments were transported by a plane primary flow which was irrotational; however, the stream surfaces of the primary flow are not necessarily at the same radius before and after the cascade. The essential result for the outlet streamwise vorticity is

$$\omega_{s_2}' = \frac{r_1}{r_2}\,\omega_{s_1}'\,\frac{\cos\beta_1}{\cos\beta_2} + \omega_{n_1}\left\{\frac{r_1}{r_2}\,\frac{\sin\beta_1}{\sin\beta_2} - \frac{W_1}{W_2}\tan\beta_2 + \frac{W_1}{P_2\cos\beta_2}\int\frac{d_s}{W_s}\right\} \quad , \tag{20}$$

where P_2 is the distance between blades at exit,

$$\int\frac{ds}{W_s} \simeq \frac{\Gamma}{W_\infty^2} \tag{21}$$

and

$$\Gamma = \frac{2\pi}{n}(r_1\,V_{r_1} - r_2\,V_{r_2}) \quad . \tag{22}$$

This is essentially the result obtained by Hawthorne and Novak [8] for a plane flow in a stationary coordinate system.

Smith [19] has presented an analysis which allows secondary vorticity to be calculated for mainstream secondary flows using the conventional axisymmetric solution as the basic flow, i.e., the flow is assumed to remain on surfaces of revolution. A major assumption of

Smith's analysis is that the distortion of those stream surfaces is not large. A relation for the secondary streamwise vorticity in the flow at the trailing edge plane was achieved with aid of the vortex laws. The expression obtained is

$$\omega_{s_2}' = \frac{1}{a}\left\{W_1 \omega_{n1}' \frac{\Gamma_{VA}}{W_\infty^2} + \frac{d\Gamma_V}{dr}\right\} \frac{dr_1}{dr_2} \tag{23}$$

where a is the distance between the exit streamlines, Γ_{VA} is the blade circulation in the actual flow, i.e., primary and secondary flow, and Γ_V is the blade circulation in the primary flow.

It is important to note that there are differences in the definition of the primary flow between Smith's theory and other theories. However, Horlock [5] shows that all approaches lead to basically the same cascade secondary flow theory.

[4] SUMMARY

Equations for the development of streamwise secondary vorticity in a blade passage for rotating systems are derived from vector equations for vorticity using intrinsic coordinates. Comparisons can be made between the secondary vorticity equations for a rotor operating at a flow coefficient near one and those which are commonly used in cascades.

The equations for a rotor operating at a flow coefficient near one show a coupling of equations for the vorticity components parallel and normal to the streamlines. In order to solve this set, the equations were nondimensionalized with respect to the characteristic parameters for this three-dimensional flow. The resulting equation for the streamwise vorticity allows for convergence of the streamlines in the radial

direction, for the effects of rotation, and for the effects of streamline curvature.

Streamwise component of vorticity is produced in a blade passage through deflection of the relative flow having a normal vorticity component. Additional effects on the streamwise vorticity occur due to rotor rotation and streamline curvature. It is interesting to note that both a relative velocity increase through the rotor and stream surface rotation decreases the amount of vorticity generated by the turning of the normal vorticity vector through the rotor.

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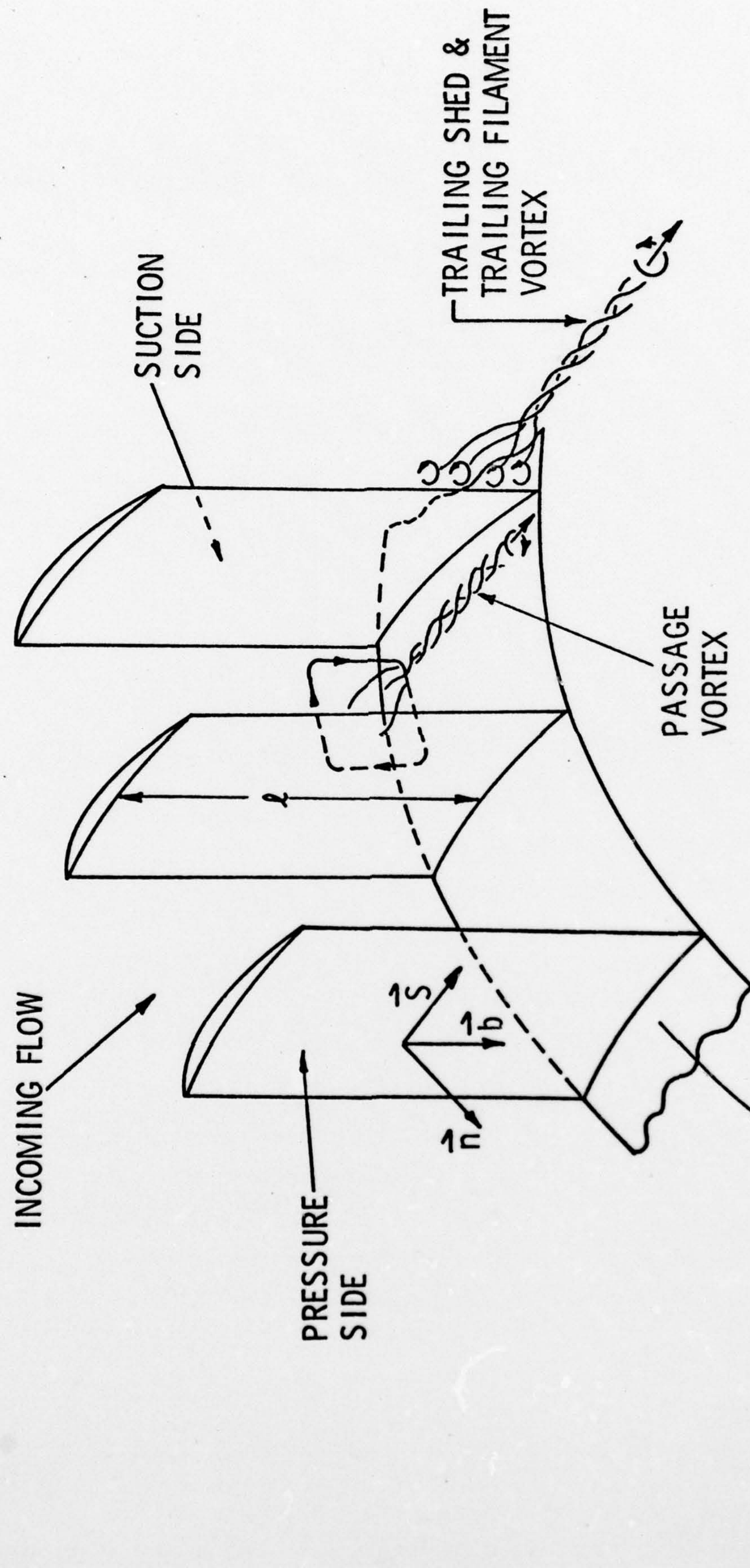

Figure 1 - Schematic of Rotor Wall Secondary Flows
(Lakshminarayana and Horlock [6])

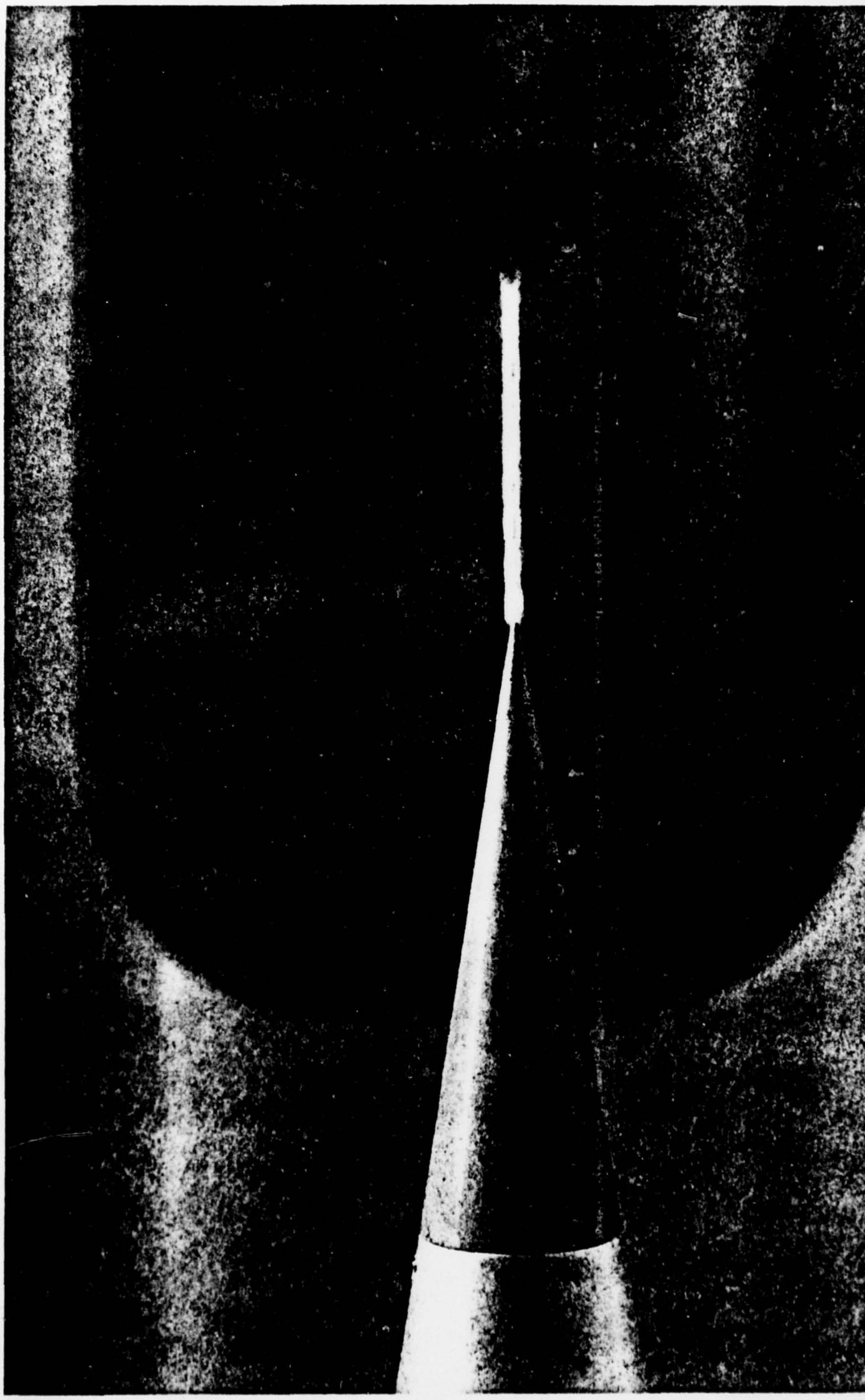

Figure 2 - Vortex Cavitation Emanating from the Conical Tip of the Vortex Generator (σ=2.9, V_∞=30 fps)

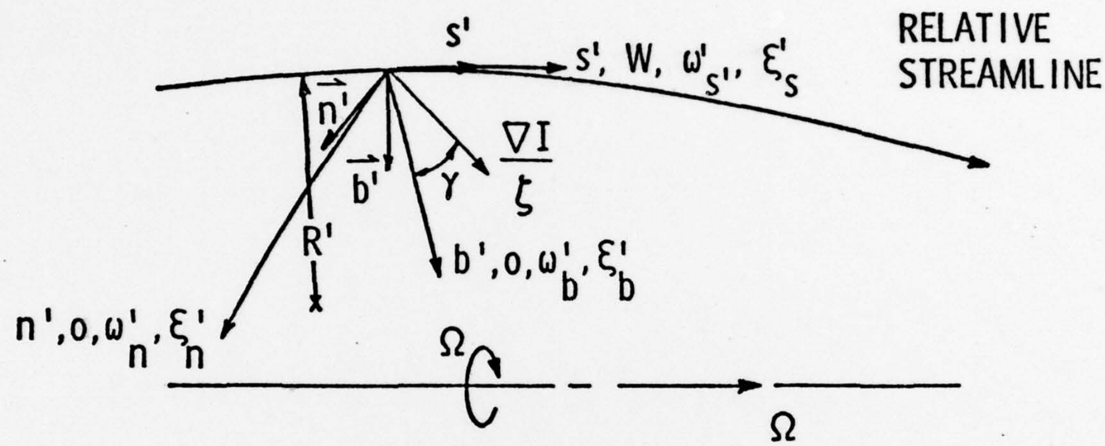

Figure 3 - Notation for Rotating Coordinate System

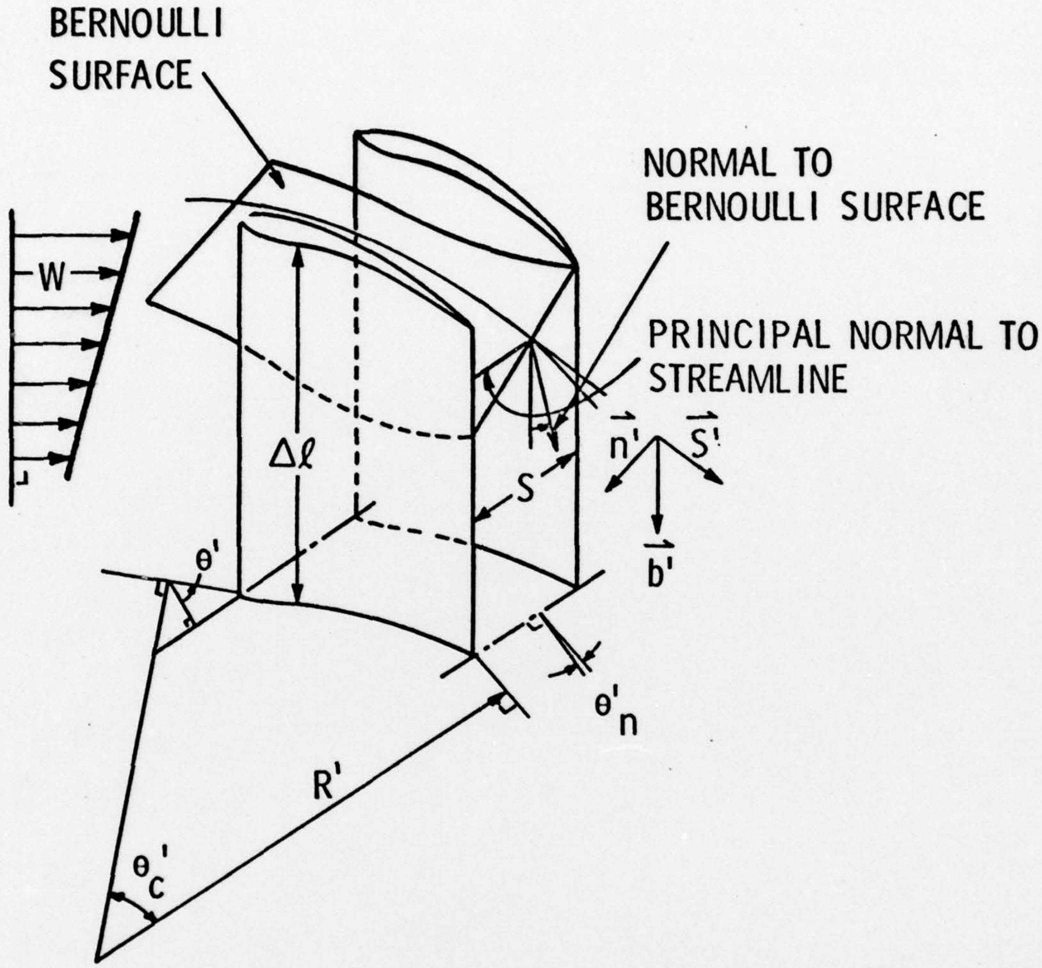

Figure 4 - Flow Through a Blade Passage